Chapter 1

REAL NUMBERS

Complete NCERT Solutions

Current syllabus: NCERT Mathematics, Class X, Reprint 2026-27
Exercises covered: 1.1 and 1.2 | Every question solved step by step

What this notebook contains

- *Prime factorisation, HCF and LCM*
- *Fundamental Theorem of Arithmetic*
- *Proofs of irrational numbers by contradiction*
- *Important exam lines and common mistakes*

Write each step clearly. In proof questions, always mention the contradiction and the final conclusion.

Important Concepts & Formula Sheet

Fundamental Theorem of Arithmetic: Every composite number can be expressed as a product of primes, and this factorisation is unique except for the order of the prime factors.

1. For two positive integers a and b:

HCF(a, b) × LCM(a, b) = a × b

This identity is guaranteed for two positive integers. Do not use it directly for three numbers.

2. Prime-factor method

- *HCF = product of the smallest powers of all common prime factors.*
- *LCM = product of the greatest powers of every prime factor present.*

3. Key theorem for irrationality proofs

If p is a prime and p divides a^2, then p divides a.

4. Standard contradiction format

Assume the given number is rational → write it as a/b in lowest form → derive that a and b have a common factor → contradiction → therefore the number is irrational.

Important meaning

Rational number: a number expressible as p/q, where p and q are integers and q ≠ 0. Irrational number: a real number that cannot be expressed in this form.

EXERCISE 1.1

Q1. Express each number as a product of its prime factors.

(i) 140

$$\begin{aligned} 140 &= 2 \times 70 \\ &= 2 \times 2 \times 35 \\ &= 2 \times 2 \times 5 \times 7 \end{aligned}$$

Answer: $140 = 2^2 \times 5 \times 7$

(ii) 156

$$\begin{aligned} 156 &= 2 \times 78 \\ &= 2 \times 2 \times 39 \\ &= 2 \times 2 \times 3 \times 13 \end{aligned}$$

Answer: $156 = 2^2 \times 3 \times 13$

(iii) 3825

$$\begin{aligned} 3825 &= 5 \times 765 \\ &= 5 \times 5 \times 153 \\ &= 5^2 \times 3 \times 51 \\ &= 3^2 \times 5^2 \times 17 \end{aligned}$$

Answer: $3825 = 3^2 \times 5^2 \times 17$

(iv) 5005

$$\begin{aligned} 5005 &= 5 \times 1001 \\ &= 5 \times 7 \times 143 \\ &= 5 \times 7 \times 11 \times 13 \end{aligned}$$

Answer: $5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

$$\begin{aligned} 7429 &= 17 \times 437 \\ &= 17 \times 19 \times 23 \end{aligned}$$

Answer: $7429 = 17 \times 19 \times 23$

Exam check: Continue division until every factor is prime. A composite factor such as 35 or 143 is not the final answer.

EXERCISE 1.1

Q2. Find the LCM and HCF and verify LCM × HCF = product of the two numbers.

(i) 26 and 91

Prime factors:

$26 = 2 \times 13$

$91 = 7 \times 13$

HCF = 13

LCM = $2 \times 7 \times 13 = 182$

Verification:

$13 \times 182 = 2366$

$26 \times 91 = 2366$

Hence verified.

(ii) 510 and 92

Prime factors:

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2^2 \times 23$

HCF = 2

LCM = $2^2 \times 3 \times 5 \times 17 \times 23 = 23{,}460$

Verification:

$2 \times 23{,}460 = 46{,}920$

$510 \times 92 = 46{,}920$

Hence verified.

(iii) 336 and 54

Prime factors:

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3^3$

HCF = $2 \times 3 = 6$

LCM = $2^4 \times 3^3 \times 7 = 3024$

Verification:

$6 \times 3024 = 18{,}144$

$336 \times 54 = 18{,}144$

Hence verified.

EXERCISE 1.1

Q3. Find the LCM and HCF by prime factorisation.

(i) 12, 15 and 21

$12 = 2^2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$

HCF = 3

LCM = $2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

17, 23 and 29 are distinct prime numbers.

HCF = 1

LCM = $17 \times 23 \times 29 = 11{,}339$

(iii) 8, 9 and 25

$8 = 2^3$; $9 = 3^2$; $25 = 5^2$

HCF = 1

LCM = $2^3 \times 3^2 \times 5^2 = 1800$

For HCF, a prime must be common to every given number. If no prime is common to all, HCF = 1.

EXERCISE 1.1

Q4. Given HCF(306, 657) = 9, find LCM(306, 657).

For two positive integers:

$HCF \times LCM$ = Product of the numbers

$9 \times LCM = 306 \times 657$

$LCM = (306 \times 657) / 9$

$= 34 \times 657$

$= 22{,}338$

Answer: LCM(306, 657) = 22,338

Q5. Check whether 6^n can end with digit 0 for any natural number n.

If 6^n ends in 0, it must be divisible by $10 = 2 \times 5$. Therefore its prime factorisation must contain both 2 and 5.

But $6^n = (2 \times 3)^n = 2^n \times 3^n$. Its only prime factors are 2 and 3; factor 5 is absent.

Answer: No natural number n makes 6^n end with 0.

Important line: A number ends with 0 only when its prime factorisation contains both 2 and 5.

EXERCISE 1.1

Q6. Explain why the given numbers are composite.

(i) $7 \times 11 \times 13 + 13$

Take 13 common:

$7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1)$

$= 13(77 + 1)$

$= 13 \times 78$

It has factors 13 and 78, both greater than 1. Therefore it is composite.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

Take 5 common:

$= 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5(1008 + 1)$

$= 5 \times 1009$

It has factors 5 and 1009, both greater than 1. Therefore it is composite.

Q7. Sonia completes a round in 18 min and Ravi in 12 min. When will they meet again at the starting point?

They will be together at the starting point after the least common multiple of their round-times.

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

$LCM = 2^2 \times 3^2 = 36$

Answer: They will meet again after 36 minutes.

EXERCISE 1.2 – Irrational Numbers

Q1. Prove that $\sqrt{5}$ is irrational.

Assume, contrary to what we want to prove, that $\sqrt{5}$ is rational.

Then $\sqrt{5} = a/b$, where a and b are coprime integers and $b \neq 0$.

Squaring both sides:

$$5 = a^2/b^2 \Rightarrow a^2 = 5b^2$$

Thus 5 divides a^2. Since 5 is prime, 5 divides a. Let $a = 5k$ for some integer k.

Substitute $a = 5k$:

$$(5k)^2 = 5b^2$$

$$25k^2 = 5b^2 \Rightarrow b^2 = 5k^2$$

Hence 5 divides b^2, so 5 divides b.

Therefore 5 is a common factor of a and b. This contradicts the assumption that a and b are coprime.

Conclusion: $\sqrt{5}$ is irrational.

Proof-writing key: state 'a and b are coprime' at the start and explicitly write the contradiction at the end.

EXERCISE 1.2

Q2. Prove that $3 + 2\sqrt{5}$ is irrational.

Assume that $3 + 2\sqrt{5}$ is rational. Let

$3 + 2\sqrt{5} = r$, where r is rational.

Rearranging:

$2\sqrt{5} = r - 3$

$\sqrt{5} = (r - 3)/2$

Since r is rational, $(r - 3)/2$ is also rational. This would make $\sqrt{5}$ rational.

But $\sqrt{5}$ is irrational. This is a contradiction.

Conclusion: $3 + 2\sqrt{5}$ is irrational.

> ***Useful rule: Rational ± irrational = irrational. A non-zero rational × irrational = irrational.***

Q3(i). Prove that $1/\sqrt{2}$ is irrational.

Assume $1/\sqrt{2}$ is rational; let $1/\sqrt{2} = r$, where $r \neq 0$ is rational. Then $\sqrt{2} = 1/r$, which is rational. But $\sqrt{2}$ is irrational. Contradiction.

Conclusion: $1/\sqrt{2}$ is irrational.

EXERCISE 1.2

Q3(ii). Prove that $7\sqrt{5}$ is irrational.

Assume $7\sqrt{5}$ is rational; let $7\sqrt{5} = r$, where r is rational.

Then $\sqrt{5} = r/7$. Since $r/7$ is rational, this would mean $\sqrt{5}$ is rational.

But $\sqrt{5}$ is irrational. This is a contradiction.

Conclusion: $7\sqrt{5}$ is irrational.

Q3(iii). Prove that $6 + \sqrt{2}$ is irrational.

Assume $6 + \sqrt{2}$ is rational; let $6 + \sqrt{2} = r$, where r is rational.

Then $\sqrt{2} = r - 6$. Since $r - 6$ is rational, this would mean $\sqrt{2}$ is rational.

But $\sqrt{2}$ is irrational. This is a contradiction.

Conclusion: $6 + \sqrt{2}$ is irrational.

Never stop after rearranging. The final two lines must identify the contradiction and state that the original number is irrational.

One-Page Revision

Must-learn results

1. Prime factorisation of a composite number is unique, except for order.

2. HCF uses smallest powers; LCM uses greatest powers.

3. For two positive integers: HCF × LCM = product of numbers.

4. If prime p divides a^2, then p divides a.

5. In contradiction proofs, begin with the opposite assumption and end by rejecting it.

Final answers at a glance

Ex. 1.1 Q1: $140 = 2^2 \times 5 \times 7$; $156 = 2^2 \times 3 \times 13$; $3825 = 3^2 \times 5^2 \times 17$; $5005 = 5 \times 7 \times 11 \times 13$; $7429 = 17 \times 19 \times 23$.

Q2: (13, 182); (2, 23,460); (6, 3024), written as (HCF, LCM).

Q3: (3, 420); (1, 11,339); (1, 1800).

Q4: 22,338. Q5: No. Q6: 13×78 and 5×1009. Q7: 36 minutes.

Ex. 1.2: $\sqrt{5}$, $3+2\sqrt{5}$, $1/\sqrt{2}$, $7\sqrt{5}$ and $6+\sqrt{2}$ are irrational.

Exam tip: Show factorisation and verification calculations. In irrationality proofs, do not write only the rule; present the contradiction in complete steps.

Source alignment: NCERT Mathematics Textbook for Class X, Chapter 1, Reprint 2026-27.